Generating Series

# Generating Series

## Ideas of Generating Series

* Encode some counting problem to the coefficient of a power series
* Use some combinatorics to get a formula for the series
* Use some algebra to determine the coefficient of the series

## Definition of generating series:

Let be a set and let be a function  
Interpretation: for any , is the weight of , ie, it's size

E.g:

Take be the set of multisets with types of element.

for , its weight is

Let be the set of objects of weight

Note is an infinite disjoint union

For each we want to determine .

This only makes sense if its finite, so we assume each of these sets is finite

![](data:image/png;base64;base64,)

Weight Function

![](data:image/png;base64;base64,iVBORw0KGgoAAAANSUhEUgAAACAAAAAgCAYAAABzenr0AAAAAXNSR0IArs4c6QAAALVJREFUWEftlMENgDAMA80msAmjwGQwCpvAKChSkUrVkjg8+glPKsdXJ82Azt/Q2R8BEAlEApGAlsCcFtXhXFiq/gtgBHAm4xXATkIsALakmQBcNb0VQLQMRG4uWheACMtCFghKo80AC0GZS3ELgBWCNmcANAiXOQvQgpD/z7Szw2puQf6CytvmZ5Yhfb1G6wyUT7gGQZt7WtBKwmX+F0D06qrVtqe3BVpd83kARAKRQCTQPYEb7jAkISBzezgAAAAASUVORK5CYII=)

A function is a **weight function**  
iff for all is finite

![](data:image/png;base64;base64,)

Generating Series

Let be a set with a weight function   
The generating series of with respect to is

![](data:image/png;base64;base64,)

Proposition:

let be a set with a weight function, then for all ,

Proof

## Sum Lemma

## Product Lemma

Let be sets with weight functions ,

Define as follows:

for , let

Then

## String Lemma

Let be a set with a weight function

such that (there are no elements of weight 0)

![](data:image/png;base64;base64,)

Why no elements of weight 0?

ex.

There are infinitely many elements of of weight 0.  
 for any finite sequene of 0s  
So is not a valid weight function on

Let

for , times = has generating series

Let

For ,

Its weight is   
Then